Ratio & Root Tests (8.5)

Ratio Test

Let $\sum_{n=1}^{\infty} a_n$ be a series w/ positive terms.

- If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$ then $\sum_{n=1}^{\infty} a_n$ converges.
- If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$ then $\sum_{n=1}^{\infty} a_n$ diverges
- (If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ then we know nothing about $\sum_{n=1}^{\infty} a_n$!)

$$\sum_{n=0}^{\infty} \frac{(2n)!}{n!\,n!}$$

Ratio Test

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$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$$= \lim_{n\to\infty} \frac{\frac{(2(n+1))!}{(n+1)!\,(n+1)!}}{\frac{(2n)!}{n!\,n!}} = \lim_{n\to\infty} \frac{(2n+2)!}{(2n)!} \cdot \frac{n!\,n!}{(n+1)!\,(n+1)!}$$

$$= \lim_{n\to\infty} \frac{\cancel{(2n)!}\,(2n+1)\overset{2}{\cancel{(2n+2)}}}{\cancel{(2n)!}} \cdot \frac{\cancel{n!}\;\cancel{n!}}{\cancel{n!}\,\cancel{(n+1)}\;\cancel{n!}\,(n+1)}$$

$$= \lim_{n\to\infty} \frac{2(2n+1)}{n+1} = \lim_{n\to\infty} \frac{4n+2}{n+1} = \lim_{n\to\infty} \frac{4\cancel{n}}{\cancel{n}} = 4$$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$ so $\sum$ ~ diverges