Calculus II Spring 2010 Worksheet 3

Your Name: **Instructor: Steven Clontz**

Draw a box around your final answer. You must show all work to receive credit.

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1. Improper Integral Convergence Tests (7.7)

   (a: Based on #50)

   Show $\int_0^\infty \frac{d\theta}{1+e^\theta}$ converges by using Direct Comparison to the convergent integral $\int_0^\infty \frac{d\theta}{e^\theta}$.

   (b: Based on #60)

   Use the Limit Comparison Test to show if $\int_{e^e}^\infty \ln(\ln x)dx$ converges or diverges. (HINT: Consider $\int_{e^e}^\infty \ln x dx$.)

   (See 7.7 #47-60 for more examples of comparison tests on improper integrals.)

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2. Finding Terms of a Sequence (8.1)

Write out the first six terms of the following sequences. (Example: $x_n = \langle 1, 3, 5, 7, 9, 11, ... \rangle$)

(a: #4) $a_n = 2 + (-1)^n$

(b: #6) $b_n = \left\langle \frac{2^n - 1}{2^n} \right\rangle_{n=1}^{\infty}$

(c: #10) $c_1 = -2$ and $c_{n+1} = \frac{n}{n+1} c_n$

(d: #12) $d_1 = 2$, $d_2 = -1$ and $d_{n+2} = \frac{d_{n+1}}{d_n}$.

3. Defining a Sequence for a list of Terms (8.1)

(a: #20) Recursively define a_n given $a_n = \langle 2, 6, 10, 14, 18, ... \rangle$.

(b:) Now explicitly define the a_n from part (a).

(c: #14) Recursively define c_n given $c_n = \langle -1, 1, -1, 1, ... \rangle$.

(d:) Now explicitly define the c_n from part (c).

4. Convergence of Sequences (8.1)

Find whether the following sequences converge or diverge. If they converge, find the limit of the sequence.

(a: #30) $a_n = \dfrac{1-n^3}{70-4n^2}$

(b: #36) $b_n = \left(-\dfrac{1}{2}\right)^n$

(c: #40) $c_n = n\pi\cos(n\pi)$

(d:) $d_n = n\pi\sin(n\pi)$

(See 8.1 #1-82 for more examples of questions concerning sequences.)

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5. Finding Infinite Sums (8.2)

Express the following infinite sums or repeating decimals as a whole number or fraction of two integers.

(a:) $\frac{7}{3} + \frac{7}{9} + \frac{7}{27} + \frac{7}{81} + \frac{7}{243} + \frac{7}{729} + ...$

(b: #6) $\frac{5}{2} + \frac{5}{6} + \frac{5}{12} + \frac{5}{20} + \frac{5}{30} + ...$ (Hint: $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k-1}$)

(c:) $0.5\overline{23} = 0.5\ 23\ 23\ 23...$

6. Evaluating Geometric and Telescoping Series (8.2)

For each of the following, either find the value of the series, or explain why it diverges.

(a: #12) $\sum_{n=0}^{\infty} \left(\frac{5}{2^n} - \frac{1}{3^n} \right)$

(b:) $\sum_{k=2}^{\infty} \frac{5k^2}{2k^2+1}$

(c: #30) $\sum_{n=1}^{\infty} \ln\left(\frac{1}{n}\right)$ (Hint: Recall $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$)

(d:) $\sum_{i=-2}^{\infty} \frac{\pi^n - 1}{e^n}$

(See 8.2 #1-40 and #51-58 for more examples of Infinite Series.)

7. Integral Test (8.3)

Do the following series converge or diverge?

(a:) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{(10+20e^n)^2}$

(b:) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k \ln k}$

(See 8.3 #1-28 for more examples of the Integral Test.)

8. Comparison Tests (8.4)

Do the following series converge or diverge?

(a: #2) $\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$

(b: #8) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2}}$

(See 8.4 #1-36 for more examples of using LCT and DCT.)

9. Ratio and Root Tests (8.5)

Do the following series converge or diverge?

(a: #4) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$

(b: #12) $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^n}$

(See 8.5 #1-26 for more examples of using the Ratio/Root Tests.)

10. Alternating Series (8.6)

Do the following series converge absolutely, converge conditionally, or diverge?

(a: #14) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$

(b: #22) $\displaystyle\sum_{n=2}^{\infty} \frac{(-2)^{n+1}}{n+5^n}$

(c: #16) $\sum_{n=1}^{\infty}(-1)^n \frac{n!}{2^n}$

(d: #30) $\sum_{n=1}^{\infty}(-5)^{-n}$

(See 8.6 #1-42 for more examples of Absolute/Conditional convergence.)